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SUBJECT: Interplanetary Navigation - An
Earth-based Multiple-Link Doppler
Tracking System* - Case 720

DATE: July 25, 1968

FROM: C. C. H. Tang

ABSTRACT

Conventional single-link Earth-based Doppler track-
ing systems alone may not meet the accuracy requirements of
planetary missions involving targeting and probe landing, mainly
because of uncertainties in the ephemerides.

It is shown that a satellite transponder orbiting a
target planet may be useful as a navigational reference. The
range-rate between the satellite and a vehicle on a mission to
the target planet can be determined by a proposed Earth-based
multiple-link Doppler tracking system. The accuracy of the state-
vector estimate of the vehicle may be improved by using the
multiple-link Doppler system and through a combined estimation
procedure which no longer involves the uncertainties of the
ephemerides and the Earth station locations.   This should pro-
vide the basis for determining the trajectories of vehicles on
planetary missions with an accuracy comparable to those on ter-
restrial missions. Areas of investigation necessary to determine
the feasibility of the proposed technique for achieving such ac-
curacy are identified.

*Disclosure to NASA as "Reportable Item No. 46" under the "New
Technology" clause.

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MEMORANDUM FOR FILE

Introduction

The space flight navigation, guidance, and control problem is basically a series of trajectory estimations (or orbit determinations) and corrective maneuvers such that the trajectory will accomplish the mission. For interplanetary missions, the navigation task can be categorized in three phases: injection, midcourse, and approach. This study concentrates on navigation during the approach phase to the target planet.

Conventional Earth-based Doppler tracking systems alone may not meet the requirements of planetary missions involving targeting or probe landing, mainly because of errors in the ephemerides. For example, during Mars oppositions, the error in distance between the Earth and Mars due to a one-sigma uncertainty of 250 km in A.U. is about 100 km, and a one-sigma uncertainty of one arc-second in angular measurements results in an error of about 300 km in position. These errors together with tracking station location errors (about 30 m) are reflected in the state-vector determination of a planetary vehicle by the conventional Earth-based Doppler tracking system.

To improve the accuracy of Earth-based interplanetary navigation by eliminating the dependence on the ephemerides a multiple-link Doppler tracking system is proposed.

Principles of Operation

For purposes of this discussion, a planetary flyby mission is shown diagrammatically in Fig. 1. It is assumed for simplicity in presentation that a satellite transponder S orbiting in a known orbit with respect to the center of the target planet is used as the navigation reference. The validity of this assumption will be

discussed later. The object of placing the satellite transponder in orbit is to provide the range-rate between the vehicle V and the satellite transponder from measurements on the Earth, not on the vehicle. To improve the estimate of the trajectory of the flyby vehicle V, it is desirable to determine the range-rate $\dot{\rho}_{SV}$ between the satellite S and the vehicle V. To this end, the following sequence of events is employed: The observation station O transmits a frequency f_1 toward the orbiting satellite and its transponder responds with a frequency f_2 toward the observation station and the vehicle. The transponder in the vehicle sees an apparent frequency f_3 and responds with two frequencies f_3 and f_4 simultaneously. The frequency f_3 varies according to the range-rate $\dot{\rho}_{SV}$, and f_4 is a fixed known frequency. There are two different Doppler counts D_3 and D_4 at the observation station because two different frequencies are emitted from the vehicle. Since the Doppler count is directly proportional to the originating source frequency, the variable unknown frequency f_3 can be determined by the relations:

$$\frac{f_3}{f_4} = \frac{D_3}{D_4} , \tag{1}$$

$$D_3 = f_3 - f_5, \tag{2}$$

$$D_4 = f_4 - f_6, \tag{3}$$

where f_5 is the apparent frequency received at the station and transmitted as frequency f_3, and

f_6 is the apparent frequency received at the station and transmitted as frequency f_4.

The solution of equations (1) through (3) yields:

$$f_3 = \frac{f_4 f_5}{f_6} . \tag{4}$$

(Note that to avoid any isolation problem in receiving and transmitting f_3 simultaneously at the vehicle f_3 can be converted to f_3', which differs from f_3 by a constant known factor and is therefore coherent with f_2. f_3' instead of f_3 can be transmitted simultaneously with f_4.)

The range-rate between the vehicle V and the spacecraft S can be calculated as:

$$\dot{\rho}_{SV} = C(1 - \frac{f_4 f_5}{f_2 f_6}),\tag{5}$$

where C is the velocity of light.

The trajectory of the vehicle with respect to the planet center can be estimated once $\dot{\rho}_{SV}$ is known.

The essence of the scheme is to determine the frequency f_3 received by the vehicle through Earth-based facilities. Note that the frequency f_3 could be determined on board the vehicle with the addition of a high precision frequency counter, but this information on frequency f_3 would have to be transmitted to the Earth for trajectory estimation. On the other hand, the direct determination of frequency f_3 on board would have less noise corruption than the Earth-based determination because of the extra noise corruption in two frequency countings. Coherent removal of the two-way satellite/Earth Doppler at the Earth station would yield the exact value of f_2 transmitted to the vehicle.

Estimation Criterion

Excluding the noise corruption in frequency counting, the expected accuracy of the trajectory estimation of the flyby vehicle depends on the assumption that the orbiting satellite is in a known orbit. If the satellite orbit is estimated by conventional Earth-based single-link Doppler measurements, the errors of the ephemerides and station locations are reflected in the state-vector estimate. Use of such an estimate as a navigation reference will not enable us to establish a better flyby vehicle trajectory estimate than that obtained by the conventional single-link scheme. However, the range-rate $\dot{\rho}_{SV}$ obtained by the multiple-link scheme enables us to improve the state-vector estimates of both the satellite and the flyby vehicle by a combined estimation procedure which no longer involves the errors of the ephemerides and Earth station locations. A brief description of the combined estimation procedure follows.

Referring to Figure 2, we have

$$\underline{d}(X_V) = \underline{r}(X_S) + \underline{\rho}(X_V, X_S), \tag{6}$$

or

$$\underline{d}(X_V) = \underline{r}(X_S) + \underline{\rho}(X),$$

where X_V represents the six components of the state-vector of the vehicle,

X_S represents the six components of the state-vector of the satellite,

X represents the twelve components X_V and X_S,

$\underline{d}$ is the vector between the center of the planet and the vehicle,

$\underline{r}$ is the vector between the center of the planet and the satellite, and

$\underline{\rho}$ is the vector between the satellite and the vehicle.

Taking the time derivative of equation (6), we have

$$\underline{\dot{d}}(X_V) = \underline{\dot{r}}(X_S) + \underline{\dot{\rho}}(X). \tag{7}$$

Let

$$\underline{\rho} = \rho_{SV}\hat{\underline{\rho}}, \tag{8}$$

where ρ_{SV} is the magnitude of the distance between the satellite and the vehicle, and

$\hat{\underline{\rho}}$ is the unit vector in the direction of $\underline{\rho}$.

Dotting Equation (7) by $\hat{\underline{\rho}}$, we obtain

$$[\underline{\dot{d}}(X_V) - \underline{\dot{r}}(X_S)] \cdot \hat{\underline{\rho}}(X) = \dot{\rho}_{SV} \tag{9}$$

The least squares estimation criterion then is[1]:

$$\sum \left\{ \left[\frac{\partial \underline{\dot{d}}(X_V)}{\partial X} - \frac{\partial \underline{\dot{r}}(X_S)}{\partial X} \right] \cdot \hat{\underline{\rho}}(X) + [\underline{\dot{d}}(X_V) - \underline{\dot{r}}(X_S)] \cdot \frac{\partial \hat{\underline{\rho}}(X)}{\partial X} \right\}$$

$$\left\{ \dot{\rho}_{SV} - [\underline{\dot{d}}(X_V) - \underline{\dot{r}}(X_S)] \cdot \hat{\underline{\rho}}(X) \right\} = 0, \tag{10}$$

where

$$\frac{\partial \hat{\underline{\rho}}(X)}{\partial X} = \frac{1}{\rho} \left[\frac{\partial \underline{d}(X_V)}{\partial X} - \frac{\partial \underline{r}(X_S)}{\partial X} \right] - \frac{1}{\rho} \left[\frac{\partial \underline{d}(X_V)}{\partial X} - \frac{\partial \underline{r}(X_S)}{\partial X} \right] \cdot \hat{\underline{\rho}}\hat{\underline{\rho}}. \tag{11}$$

The classical differential correction method[1] using the least squares estimation criterion (10) yields the best estimates of both X_V and X_S.

Through the use of the multiple-link Doppler tracking and the combined estimation procedure the accuracy of the state-vector of the orbiting satellite may also be improved. Thus, use of a planetary orbiter as the navigation reference may permit improving the accuracy of trajectory estimation of vehicles on missions to the planet. The improvement is expected since the errors of the ephemerides and Earth station locations are no longer involved in the estimation procedure. The reference satellite does not require a prescribed ideal orbit, and as long as it is inserted in any orbit, it can be used for navigational purposes. This is an advantage since the requirements for inserting a planetary orbiter in any orbit are not as stringent as those of delivering vehicles on missions involving targeting and probe landing on a planet. Preliminary or initial state-vector estimates of vehicles on planetary missions can be obtained by the conventional single-link Doppler system described in reference 1.

Discussion

The proposed multiple-link Doppler tracking is a scheme for improving estimation accuracy and is not a scheme for redundancy because it depends upon the information derived by the single-link Doppler tracking. Both redundancy and improved accuracy could be obtained by on-board frequency counting and orbit determination, which would increase the weight of the vehicle.

To avoid unknown corrections of radio waves due to the planetary atmosphere and ionosphere and to reduce the effect of higher order harmonics of the gravitational potential of the planet on the satellite, the satellite should be inserted high above the atmosphere and ionosphere of the planet.

From the systems point of view, the improvement in the accuracy of interplanetary trajectory estimation is obtained at the expense of providing a planetary orbiter in advance. However, for purposes of initial comprehensive photography, occultation experiments, and other science studies, a precursory orbiter would be invaluable. Such a precursory orbiter can be designed to incorporate the features required to operate as a navigation reference.

The following items should be investigated to determine the feasibility of the proposed multiple-link Earth-based Doppler system:

1. The frequency stability and accuracy required for oscillator of f_4 to obtain the prescribed accuracy in f_3,

2. The extent of modification of ground equipment necessary to count both f_5 and f_6 simultaneously (errors due to atmospheric and ionospheric refractions are reduced),

3. Required sizes of the transmitting antenna on the orbiter and the receiving antenna on the vehicle and associated power requirements,

4. Weight and power requirements comparison between the stabilized oscillator of f_4 and on-board equipment required for precision frequency counting,

5. Clock accuracy required in time tagging either f_3 or f_4,

6. The ranges of limits by which the preliminary estimates of the state-vectors of both the satellite and the vehicle can be in error without failing to obtain the best estimates.

Conclusion

Conventional single-link Earth-based Doppler tracking systems alone may not meet the accuracy requirements of planetary missions involving targeting and probe landing, mainly because of uncertainties in the ephemerides.

It has been shown that a satellite transponder orbiting a target planet may be useful as a navigational reference. The range-rate between the satellite and a vehicle on a mission to the target planet can be determined by the proposed Earth-based multiple-link Doppler tracking system. The accuracy of the state-vector estimate of the vehicle may be improved by using the multiple-link Doppler system and through a combined estimation procedure which no longer involves the uncertainties of the ephemerides and the Earth station locations. This should provide the basis for determining the trajectories of planetary vehicles with

an accuracy comparable to those of terrestrial vehicles. Areas
of investigation necessary to determine the feasibility of the
proposed technique for achieving such accuracy are listed at the
end of the last section.

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REFERENCE

[1] C. C. H. Tang and C. L. Greer, "Determination of Orbits of Planetary Artificial Satellites and Planetary Gravitational Fields," TR-68-720-1, Bellcomm, Inc., May 24, 1968.
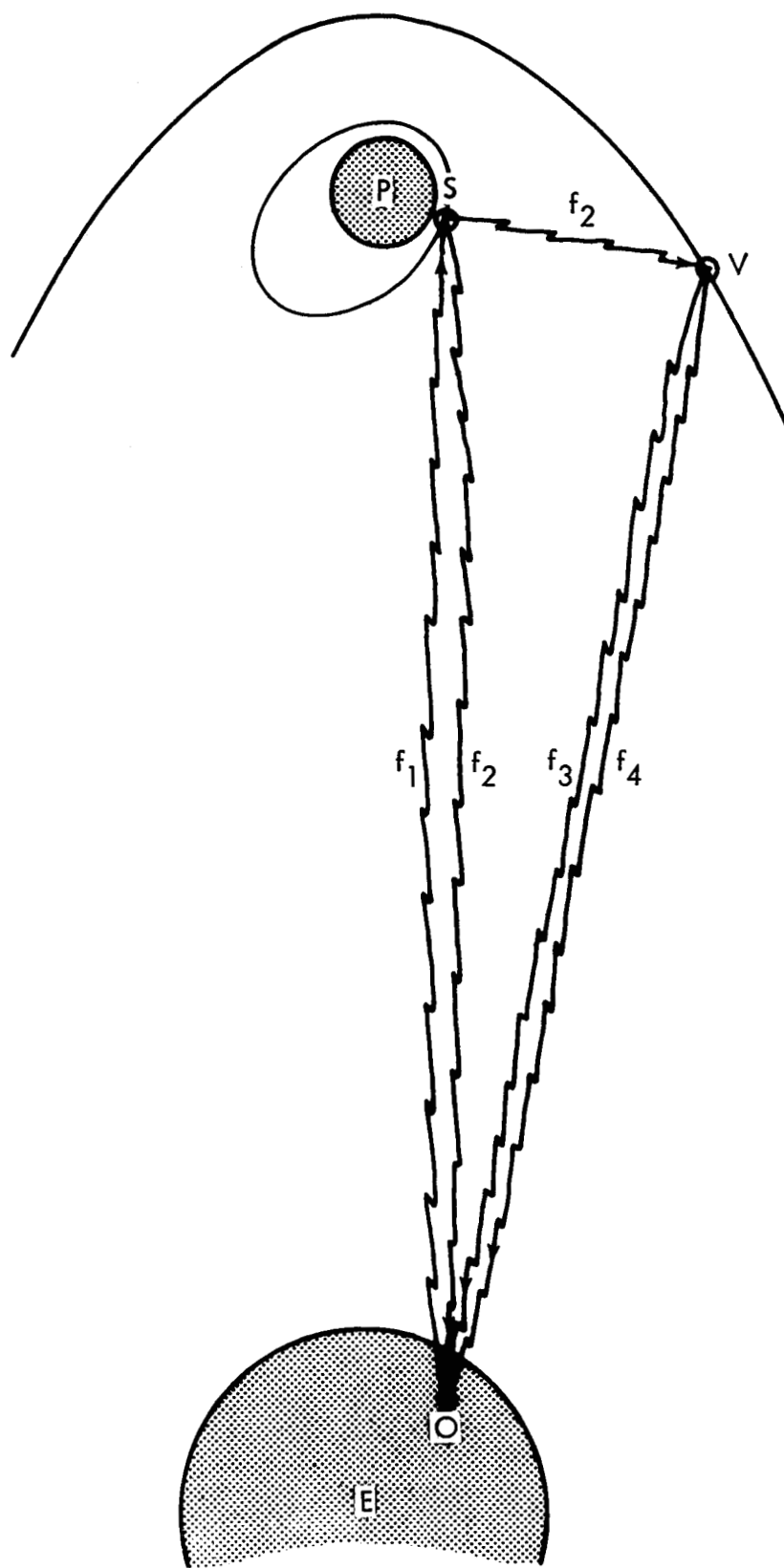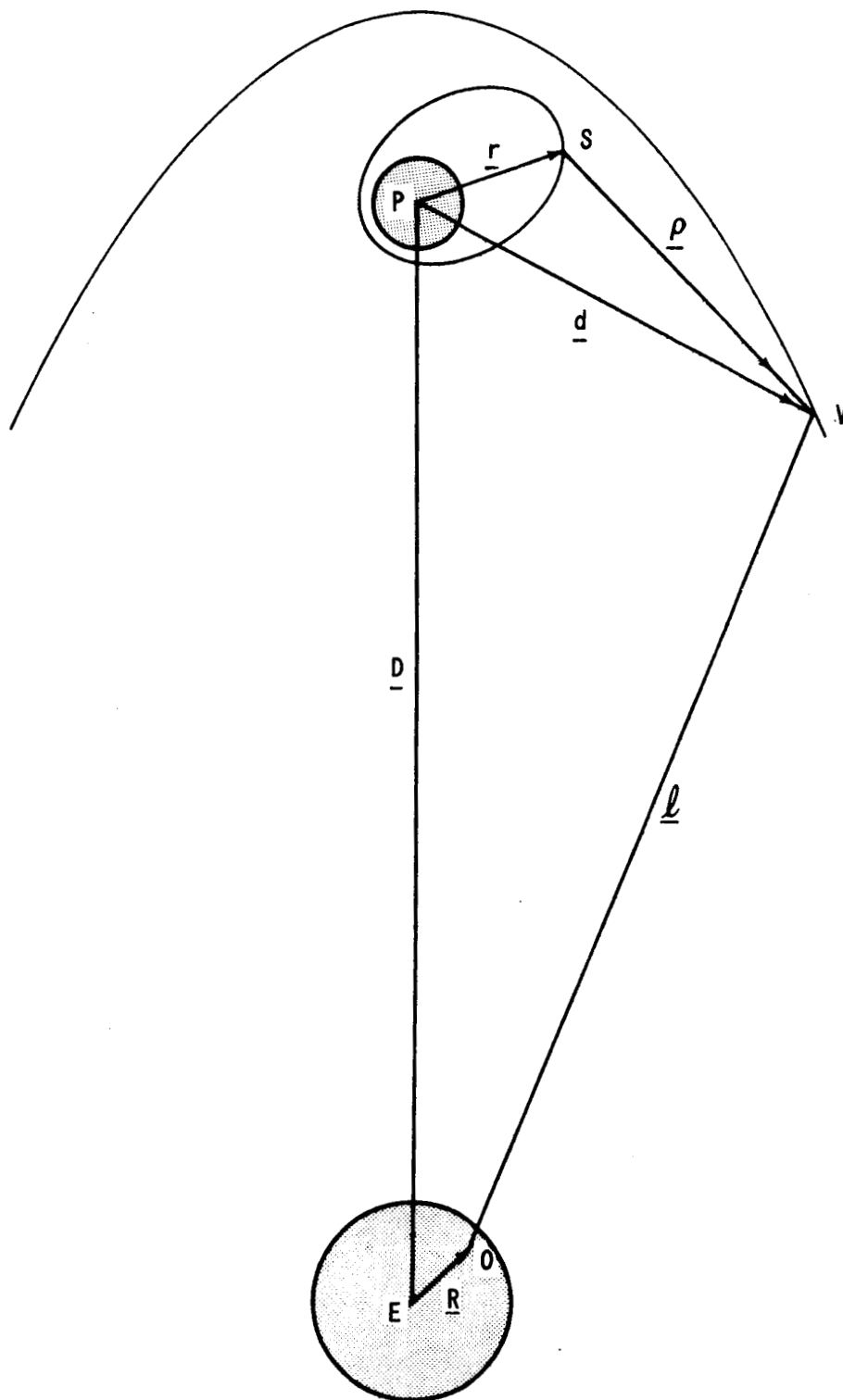
FIGURE 1 – TRACKING GEOMETRY

FIGURE 2 - A PLANETARY SATELLITE AS A NAVIGATIONAL REFERENCE

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